Strategyproofness and Monotone Allocation of Auction in Social Networks

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Auction in Social Networks: Motivation

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- Informed bidders submit bids and compete for the items.
- The auctioneer determines who get the items (allocation), and how much for charge (payment).

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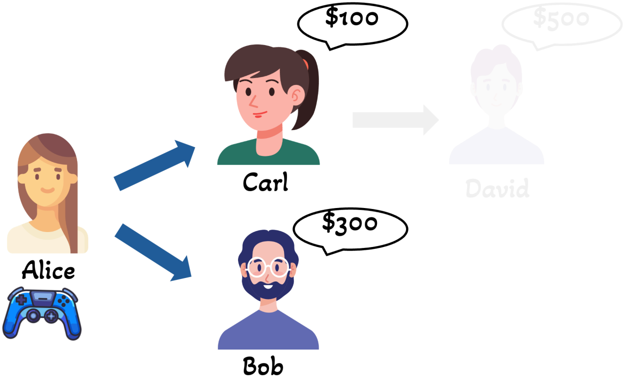
A competitive market is more desirable!

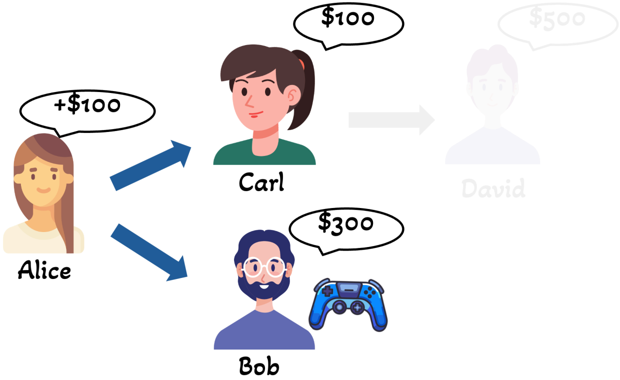
Incentivize bidders inviting new potential bidders via social interactions!

Auction in Social Networks: Model [LHZZ17]



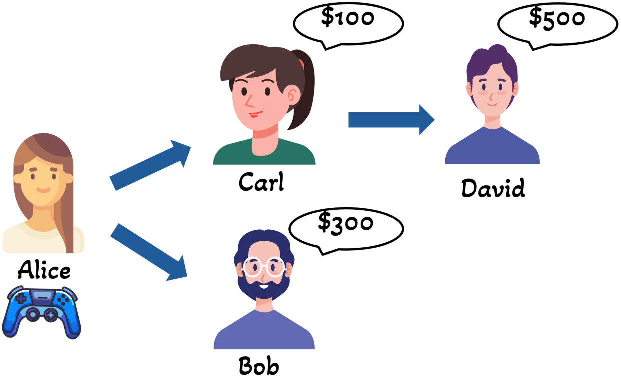
Auction with two bidders.

Auction in Social Networks: Model [LHZZ17]



Vickrey Auction

Auction in Social Networks: Model [LHZZ17]



Auction via Social Networks

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- **Efficiency**: Allocation rules that maximize social welfare.
- **Budget-balance**: no deficit for the seller.

Challenges

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  single-item $\longrightarrow$ multi-unit with unit demand.
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- (☹) **Auction in Social Networks**
  single-item $\xrightarrow{?}$ multi-unit with unit demand.
  Non-trivial!
- Some early works fail strategyproofness in multi-unit network auction with single-unit demand [ZLX+18, KBT+20].

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Missing designing principles beyond single-item settings!

Our Results

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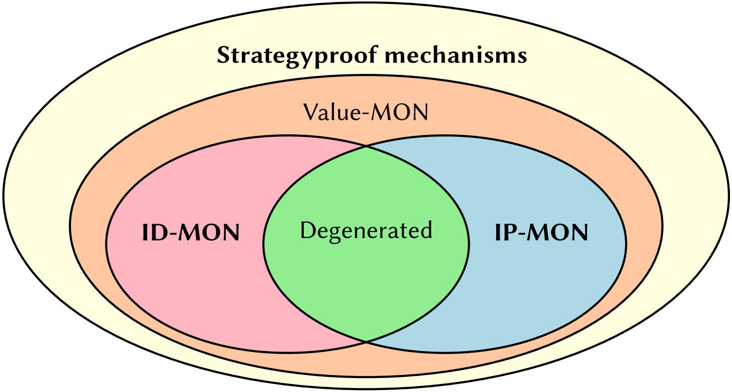

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- Two typical types of monotone allocation rules.
  - Invitation-Depressed Monotonicity (ID-MON).
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- Two typical types of monotone allocation rules.
  - Invitation-Depressed Monotonicity (ID-MON).
  - Invitation-Promoted Monotonicity (IP-MON).
- Payment rules.
  - Revenue-maximization payment rules.
  - Solvable in polynomial time by binary search.

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- Revisit the DNA-MU mechanism [KBT+20] and fix the non-strategyproofness issue.
- Refine VCG mechanism and prove the revenue upper bound for efficient allocation.
- Mechanism design for network auction with single-minded bidders.

Thank you for your attention! Q & A

Poster Area 2 - Line F 27-42. (16:30 - 18:00)

[BK94] Jeremy I Bulow and Paul D Klemperer. Auctions vs. negotiations, 1994.

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[LHZ20] Bin Li, Dong Hao, and Dengji Zhao. Incentive-compatible diffusion auctions. *arXiv preprint arXiv:2001.06975*, 2020.

[LHZZ17] Bin Li, Dong Hao, Dengji Zhao, and Tao Zhou. Mechanism design in social networks. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 31, 2017.

[Mye81] Roger B Myerson. Optimal auction design. *Mathematics of operations research*, 6(1):58–73, 1981.

[ZLX+18] Dengji Zhao, Bin Li, Junping Xu, Dong Hao, and Nicholas R Jennings. Selling multiple items via social networks. In *Proceedings of the 17th International*

Conference on Autonomous Agents and MultiAgent Systems, pages 68–76, 2018.

Auction in Social Networks: Model [LHZZ17]

- A set of N agents.
- A set of $\mathcal{K}$ items.
- A market $G = (N \cup \{s\}, E)$ with seller s.
- Each agent i has private information $t_i = (v_i, r_i)$, where v_i: valuation; r_i: neighbor set $r_i = \{j \mid (i, j) \in E\}$.
- Mechanism $\mathcal{M} = (f, p)$ with allocation f and payment p.
- Quasi-linear utility function $u_i = f_i \cdot v_i - p_i$.
- **Social welfare**: $\mathrm{SW}^{\mathcal{M}}(\mathbf{t}) = \sum_{i \in N} f_i \cdot v_i$.
- **Revenue**: $\mathrm{Rev}^{\mathcal{M}}(\mathbf{t}) = \sum_{i \in N} p_i$.

[1]

[1] Generated by Gemini2.5 Flash.

Axioms for strategyproofness

Myerson's Lemma [Mye81]

A mechanism $\mathcal{M} = (f, p)$ in a single-parameter domain is incentive compatible if and only if the following conditions hold:

- Allocation f is value-monotonic.
- Every winning bidder pays the critical value:

$$v^* = \inf_{v_i : f_i(v_i, v_{-i}) = 1} v_i$$

Axioms for strategyproofness

IC for Single-item Network Auction [LHZ20]

A mechanism $\mathcal{M} = (f, p)$ for single-item network auction is IC & IR if and only if the following conditions hold:

- Allocation f is value-monotonic.
- Decomposing payment as $p_i = f_i\tilde{p}_i + (1 - f_i)\bar{p}_i$. winning payment $\tilde{p}_i$ and losing payment $\bar{p}_i$ are bid-independent.
- Under truthful referral r_i, difference between $\tilde{p}_i$ and $\bar{p}_i$ is the critical value.

$$\tilde{p}(r_i) - \bar{p}(r_i) = v^*(r_i)$$

- Payment is referral-monotonic: $\forall\, r_i^1, r_i^2 \subseteq r_i, r_i^1 \subseteq r_i^2$,

$$\tilde{p}_i(r_i^1) \geq \tilde{p}_i(r_i^2).$$

Invitation-Depressed (ID) Monotonicity

Definition (Invitation-Depressed (ID) Partial Ordering $\succeq_{\mathcal{D}}$)

Given bidder i's two types: $t_i^1 = (v_i^1, r_i^1)$ and $t_i^2 = (v_i^2, r_i^2)$, define $t_i^1 \succeq_{\mathcal{D}} t_i^2$ if $v_i^1 \geq v_i^2$ and $r_i^1 \subseteq r_i^2$.

Definition (Invitation-Depressed (ID) Monotonicity)

Given allocation f, if $f_i(t_i, \mathbf{t}'_{-i}) = 1$ implies $\forall\, t'_i \succeq_{\mathcal{D}} t_i, f_i(t'_i, \mathbf{t}'_{-i}) = 1$, then f is ID-MON.

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Hint: Higher bid & fewer invitation $\Rightarrow$ better allocation.

Invitation-Depressed (ID) Monotonicity

Theorem 1

Given any ID-MON allocation f, the payment rule

$$p^* = \{p_i^* = v_i^*(\emptyset) - (1 - f_i)v_i^*(r_i)\}_{i \in N}$$

maximizes the revenue and mechanism $\mathcal{M} = (f, p^*)$ is strategyproof.

Invitation-Promoted (IP) Monotonicity

Definition (Invitation-Promoted (IP) Partial Ordering $\succeq_{\mathcal{P}}$)

Given bidder i's two types: $t_i^1 = (v_i^1, r_i^1)$ and $t_i^2 = (v_i^2, r_i^2)$, define $t_i^1 \succeq_{\mathcal{P}} t_i^2$ if $v_i^1 \geq v_i^2$ and $r_i^2 \subseteq r_i^1$.

Definition (Invitation-Promoted (IP) Monotonicity)

Given allocation f, if $f_i(t_i, \mathbf{t}'_{-i}) = 1$ implies $\forall\, t'_i \succeq_{\mathcal{P}} t_i, f_i(t'_i, \mathbf{t}'_{-i}) = 1$, then f is invitation-depressed monotone.

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Hint: Higher bid & more invitation $\Rightarrow$ better allocation.

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Definition (Invitation-Promoted (IP) Monotonicity)

Given allocation f, if $f_i(t_i, \mathbf{t}'_{-i}) = 1$ implies $\forall\, t'_i \succeq_{\mathcal{P}} t_i, f_i(t'_i, \mathbf{t}'_{-i}) = 1$, then f is IP-MON.

Invitation-Promoted (IP) Monotonicity

Theorem 2

Given one IP monotone allocation f, the payment rule

$$p^* = \{p_i^* = f_i v_i^*(r_i)\}_{i \in N}$$

maximizes the revenue and mechanism $\mathcal{M} = (f, p^*)$ is strategyproof.

Proof Sketch of Theorem 1

Revenue: $Rev^{\mathcal{M}} = \sum_{i:f_i=1} \tilde{p}_i(r_i) + \sum_{i:f_i=0} \bar{p}_i(r_i)$
IC condition for critical value: $\tilde{p}_i(r_i) - \bar{p}_i(r_i) = v_i^*(r_i)$
Rewrite $Rev^{\mathcal{M}} = \sum_{i \in W} v_i^*(r_i) + \sum_{i \in N} \bar{p}_i(r_i)$
Given an allocation, critical value is determined.
Optimize revenue $\Rightarrow$ optimize $\bar{p}_i(r_i)$.

$$
\begin{aligned}
& \max \bar{p}_i(r_i) \\
& s.t.\ \tilde{p}_i(r_i) = \bar{p}_i(r_i) + v_i^*(r_i) \\
& \quad \forall r_i' \subseteq r_i, \tilde{p}_i(r_i) \leq \tilde{p}_i(r_i') \\
& \quad \forall r_i' \subseteq r_i, \bar{p}_i(r_i) \leq \bar{p}_i(r_i') \\
& \quad v_i^*(r_i) \geq v_i^*(r_i') \geq \cdots \geq v_i^*(\emptyset) \text{ (ID-MON)} \\
& \quad \bar{p}_i(\emptyset) \leq 0
\end{aligned}
$$

Solution:

$$
\begin{aligned}
\tilde{p}_i(r_i) &= v_i^*(\emptyset) \\
\bar{p}_i(r_i) &= v_i^*(\emptyset) - v_i^*(r_i)
\end{aligned}
$$

Proof Sketch of Theorem 2

Revenue: $Rev^{\mathcal{M}} = \sum_{i:f_i=1} \tilde{p}_i(r_i) + \sum_{i:f_i=0} \bar{p}_i(r_i)$
IC condition for critical value: $\tilde{p}_i(r_i) - \bar{p}_i(r_i) = v_i^*(r_i)$
Rewrite $Rev^{\mathcal{M}} = \sum_{i \in W} v_i^*(r_i) + \sum_{i \in N} \bar{p}_i(r_i)$
Given an allocation, critical value is determined.
Maximize revenue $Rev^{\mathcal{M}} \Rightarrow$ maximize $\bar{p}_i(r_i)$.

$$\begin{aligned}
&\max \bar{p}_i(r_i) \\
&\quad s.t.\ \tilde{p}_i(r_i) = \bar{p}_i(r_i) + v_i^*(r_i) \\
&\qquad \forall r_i' \subseteq r_i, \tilde{p}_i(r_i) \le \tilde{p}_i(r_i') \\
&\qquad \forall r_i' \subseteq r_i, \bar{p}_i(r_i) \le \bar{p}_i(r_i') \\
&\qquad v_i^*(\emptyset) \ge v_i^*(r_i') \ge \cdots \ge v_i^*(r_i) \text{ (IP-MON)} \\
&\qquad \bar{p}_i(\emptyset) \le 0
\end{aligned}$$

Solution:

$$\tilde{p}_i(r_i) = v_i^*(r_i)$$
$$\bar{p}_i(r_i) = 0$$