Strategyproofness and Monotone Allocation of Auction in Social Networks

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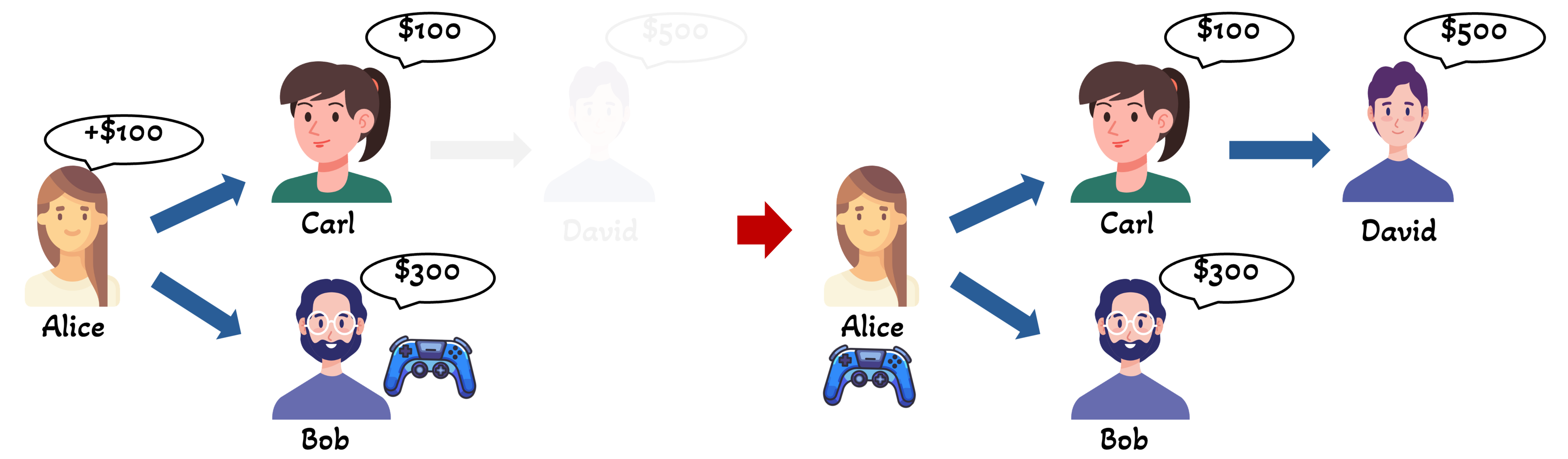
Background

"An auction with N+1 bidders is better in expected revenue than any negotiation with N bidders." [BK94]

A competitive market is more desirable!



Vickrey Auction ***Auction in Social Networks***

Model

- *A set of N agents;*
- *A set of K items;*
- *A market $G = (N \cup \{s\}, E)$ with seller s;*
- *Agent i's type: $t_i = (v_i, r_i)$, v_i: valuation, r_i: neighbor set;*
- *Mechanism $\mathcal{M} = (f, p)$*
- *Utility function $u_i = f_i \cdot v_i - p_i$;*
- ***Social Welfare***: $\mathrm{SW}^{\mathcal{M}}(\boldsymbol{t}) = \sum_{i \in N} f_i \cdot v_i$;
- ***Revenue***: $\mathrm{Rev}^{\mathcal{M}}(\boldsymbol{t}) = \sum_{i \in N} p_i$.

Axioms for Network Auction

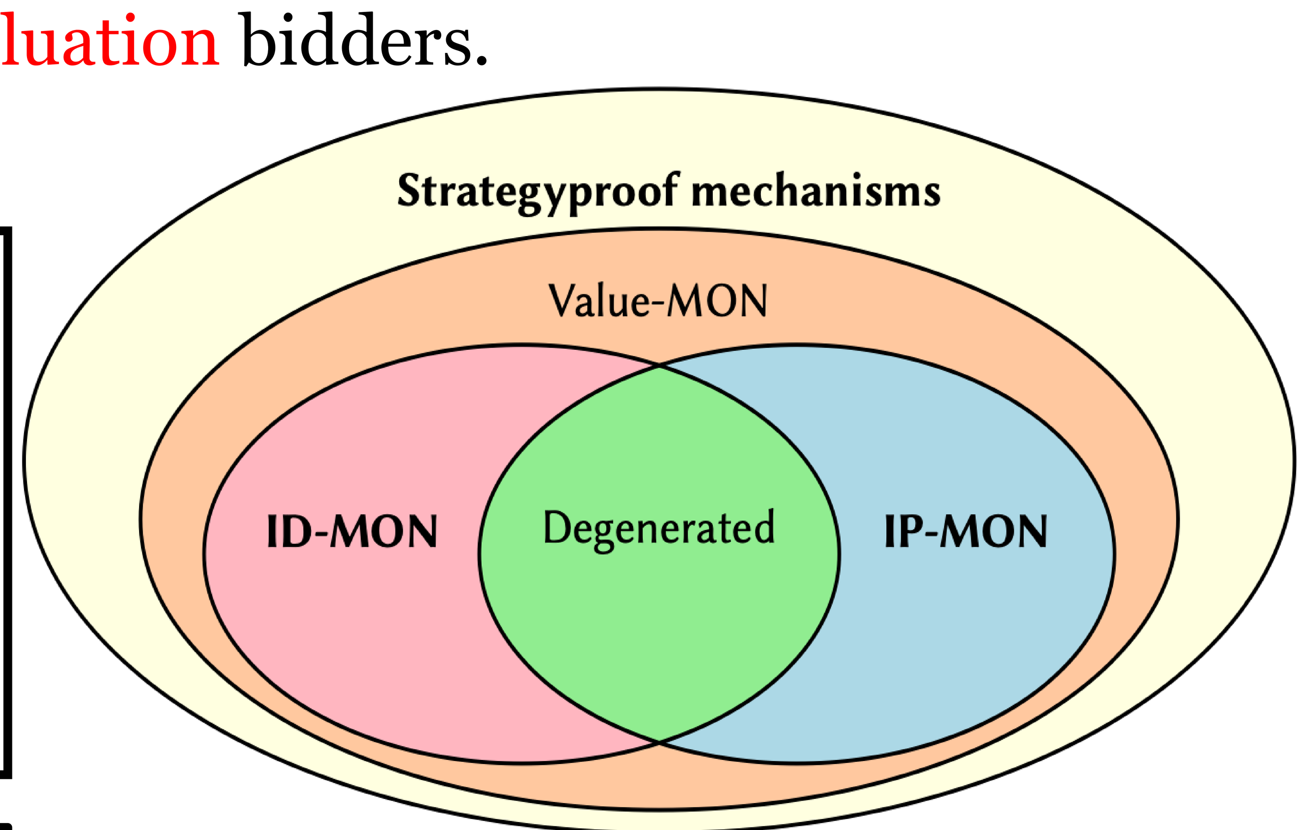
- ***Individual Rationality (IR):*** *No deficit for bidders under truthful bidding.*
- ***Strategyproofness (SP):*** *Truthfully reporting valuation and inviting all the neighbors is the dominate strategy.*
- ***Efficient (EF):*** *Maximizing social welfare.*
- ***(Weakly) Budget-balanced (WBB):*** *No deficit for the seller.*

IR & SP Network Auction [LHZ20]

- *Allocation f is value-monotone.*
- *Let $p_i = f_i \cdot \hat{p}_i + (1 - f_i) \cdot \bar{p}_i$. $\hat{p}_i$ and $\bar{p}_i$ are bid-independent.*
- *For agent i with r_i, difference between $\hat{p}_i$ and $\bar{p}_i$ is the critical value, i.e., $\hat{p}_i - \bar{p}_i = v_i^*(r_i)$.*
- *$\hat{p}_i$ and $\bar{p}_i$ are invitational-monotonic, i.e., $\forall\, r_i^1, r_i^2 \subseteq r_i, r_i^1 \subseteq r_i^2, \hat{p}_i(r_i^1) \geq \hat{p}_i(r_i^2), \bar{p}_i(r_i^1) \geq \bar{p}_i(r_i^2)$.*

Our Results

Setting: 0-1 deterministic mechanisms with single-parameterized valuation bidders.
(e.g., Single-item, multi-unit with unit-demand, single-minded, ...)



Invitational-Depressed Monotonicity (ID-MON)

Let $t_i^1 = (v_i^1, r_i^1), t_i^2 = (v_i^2, r_i^2)$, $t_i^1 \succcurlyeq_D t_i^2$ if $v_i^1 \geq v_i^2$ and $r_i^1 \subseteq r_i^2$.
f is ID-MON if $\forall\, i \in N, f_i(t_i, \boldsymbol{t}'_{-i}) \geq f_i(t'_i, \boldsymbol{t}'_{-i}), \forall\, t_i \succcurlyeq_D t'_i$.
Given ID-MON allocation f, let $\boldsymbol{p}^* = \{\boldsymbol{p_i} = \boldsymbol{v_i^*}(\emptyset) - (\boldsymbol{1} - \boldsymbol{f_i})\boldsymbol{v_i^*}(\boldsymbol{r_i})\}$.
$\mathcal{M} = (f, p^*)$ is IR and SP and for any IR and SP $\mathcal{M}' = (f, p')$,
$\mathrm{Rev}^{\mathcal{M}}(\boldsymbol{t}) \geq \mathrm{Rev}^{\mathcal{M}'}(\boldsymbol{t})$.

Invitational-Promoted Monotonicity (IP-MON)

Let $t_i^1 = (v_i^1, r_i^1), t_i^2 = (v_i^2, r_i^2)$, $t_i^1 \succcurlyeq_P t_i^2$ if $v_i^1 \geq v_i^2$ and $r_i^2 \subseteq r_i^1$.
f is IP-MON if $\forall\, i \in N, f_i(t_i, \boldsymbol{t}'_{-i}) \geq f_i(t'_i, \boldsymbol{t}'_{-i}), \forall\, t_i \succcurlyeq_P t'_i$.
Given IP-MON allocation f, let $\boldsymbol{p}^* = \{\boldsymbol{p_i} = \boldsymbol{f_i} \cdot \boldsymbol{v_i^*}(\boldsymbol{r_i})\}$.
$\mathcal{M} = (f, p^*)$ is IR and SP and for any IR and SP $\mathcal{M}' = (f, p')$,
$\mathrm{Rev}^{\mathcal{M}}(\boldsymbol{t}) \geq \mathrm{Rev}^{\mathcal{M}'}(\boldsymbol{t})$.

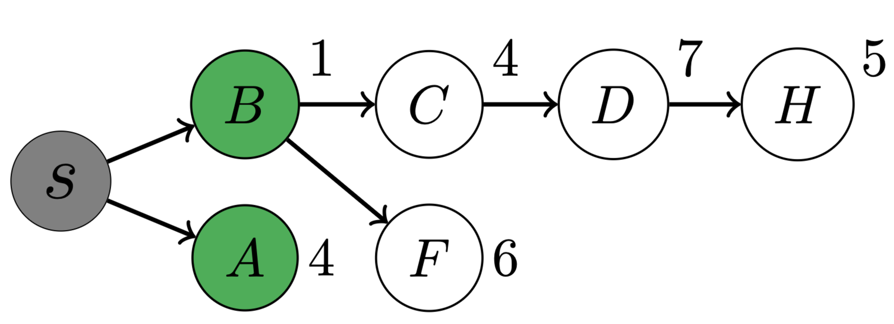
Algorithm 1 DNA-MU Mechanism [Kawasaki *et al.*, 2020]

Input: $G = (N \cup \{s\}, E), \theta, \mathcal{K}$;
Output: Allocation f, payment p;
Initialize ordering $\mathcal{O} \leftarrow \mathtt{BFS}(G, s)$;
Create Invitational-Domination Tree (IDT) T;
Initialize $k \leftarrow |\mathcal{K}|, W \leftarrow \emptyset$;
for i in $\mathcal{O}$ **do**
  $T_i \leftarrow$ Sub-Tree rooted by i in T;
  if $v_i \geq v^k(N \setminus (T_i \cup W))$ **then**
    $f_i \leftarrow 1, p_i \leftarrow v^k(N \setminus (T_i \cup W))$;
    Update $k \leftarrow k - 1, W \leftarrow W \cup \{i\}$;
  end if
end for
Return f, p.

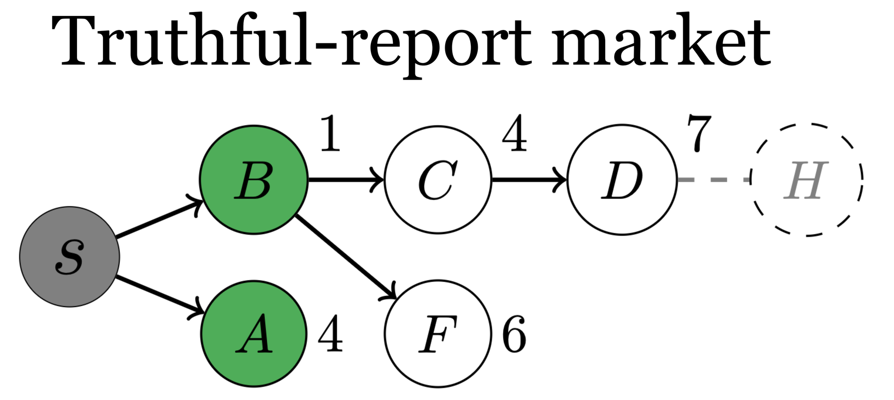
(i). IR & SP mechanisms boil down to find ID/IP-MON allocation.
(ii). Revenue-maximization payment is solvable in polynomial time.
(iii). All existing IR and SP mechanisms satisfy ID or IP-MON.

Algorithm 4 DNA-MU-Refined (DNA-MU-R) Mechanism

Input: $G = (N \cup \{s\}, E), \theta, \mathcal{K}$;
Output: Allocation f, payment p;
Initialize order $\mathcal{O} \leftarrow \mathtt{BFS}(G, s)$;
Create Invitational-Domination Tree (IDT) T;
Initialize $k \leftarrow |\mathcal{K}|, W \leftarrow \emptyset$;
for i in $\mathcal{O}$ **do**
  $T_i \leftarrow$ Sub-Tree rooted by i in T;
  if $v_i \geq v^k(N \setminus T_i)$ **then**
    $f_i \leftarrow 1, p_i \leftarrow v_i^*(r_i)$;
    Update $W \leftarrow W \cup \{i\}$;
  end if
end for
Return f, p.



Truthful-report market



D misreports $\emptyset$

$\mathcal{M}$	Market	Winner	Payment
DNA-MU	truthful	B, F, C	$p_B = 0$, $p_F = 5, p_C = 4$
	misreport	A, B, D	$p_A = 4$, $p_B = 0, p_D = 6$
DNA-MU-R	truthful	B, F, C	$p_B = 0$, $p_F = 4, p_C = 1$
	misreport	A, B, F	$p_A = 4$, $p_B = 0, p_F = 4$

Reference

[BK94] Jeremy I Bulow and Paul D Klemperer. *Auctions vs. negotiations*, 1994.
[LHZ20] Bin Li, Dong Hao, and Dengji Zhao. *Incentive-compatible diffusion auctions.* IJCAI 2020.
[Kawasaki et al. 2020] Takehiro Kawasaki, Nathanaël Barrot, Seiji Takanashi, Taiki Todo, and Makoto Yokoo. *Strategy-proof and non-wasteful multi-unit auction via social network.* AAAI 2020.