Minimizing Inequity in Facility Location Games

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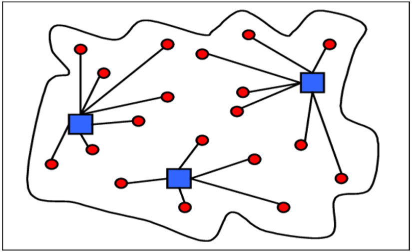
Facility Location Problem



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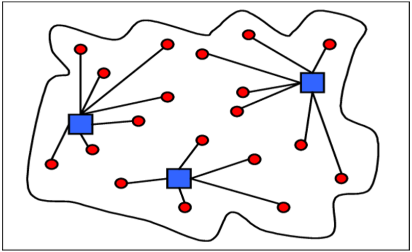

Determining the optimal locations for facilities to minimize travel costs when serving agents.

Facility Location Games

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Design mechanisms to incentivize agents truthfully reporting the location.

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Invited review

Equity measurement in facility location analysis: A review and framework

Michael T. Marsh, David A. Schilling

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"Maximum Group Effect is the earliest and most frequently used measure that has an equity component."

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- Given location subset Y, the cost incurred by agent i is defined as $c_i(Y, x_i) = \min_{y \in Y} |y - x_i|$.

Model

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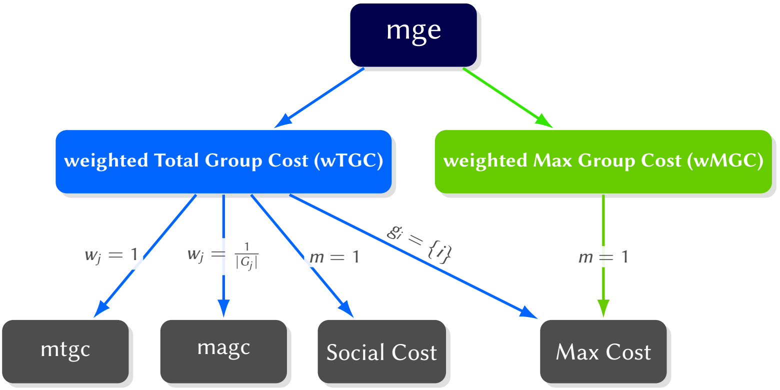
$$E_j = \begin{cases} \sum_{i \in G_j} c(f(\boldsymbol{\theta}), x_i) & \text{weighted Total Group Cost} \\ \max_{i \in G_j} c(f(\boldsymbol{\theta}), x_i) & \text{weighted Maximum Group Cost} \end{cases}$$

Our Results

Capturing Fairness Through Generalized Metrics.



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- Strategyproofness: A mechanism f is strategyproof if, for any agent i with θ_i, consider any misreported location $x'_i \in \mathbb{R}$ and fixing $\boldsymbol{\theta}'_{-i}$. We have:

$$c(f((x_i, g_i), \boldsymbol{\theta}'_{-i}), x_i) \leq c(f((x'_i, g_i), \boldsymbol{\theta}'_{-i}), x_i).$$

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- Approximation Ratio: For any mechanism f, the approximation ratio is:

$$\rho = \sup_{\boldsymbol{\theta} \in \Theta^n} \frac{\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta}))}{\text{mge}(\boldsymbol{\theta}, \text{OPT}(\boldsymbol{\theta}))},$$

where $\text{OPT}(\boldsymbol{\theta})$ is the optimal placement that minimizes mge.

Our Results

Unified Mechanisms with Tight Approximation Guarantee.

Setting	Objectives	Mechanisms	Bounds
$k=1$	wTGC	BALANCED	2
	wMGC	Major-Phantom	2
$k=2$	wTGC	EndPoints	$1+(n-2)\frac{w_{\max}}{w_{\min}}$
	wMGC	EndPoints	$1+\frac{w_{\max}}{w_{\min}}$
$k\geq 3$	General	/	∞

(All listed bounds are tight. Gray shading denotes the contributions of this work.)

Single-Facility: weighted Total Group Cost

Interpretation of Maximum Group Effect

$$\mathrm{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = \max_{j \in [m]} w_j \cdot E_j = \max_{j \in [m]} w_j \cdot \sum_{i \in G_j} c(f(\boldsymbol{\theta}), x_i).$$

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BALANCED Mechanism

Input: Agent profile $\boldsymbol{\theta}$, group weights $\{w_j\}_{j\in[m]}$.

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BALANCED Mechanism

Input: Agent profile $\boldsymbol{\theta}$, group weights $\{w_j\}_{j \in [m]}$.

1. $L_j(y) =$ # of agents in group j whose location is on the left of (or at) location y. (*non-decreasing*)

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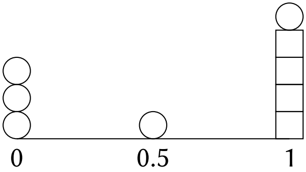
3. Place the facility at the leftmost location y such that $\max_{j \in [m]} w_j L_j(y) \geq \max_{j' \in [m]} w_{j'} R_{j'}(y)$.

Balanced Mechanism: Example

Consider an instance with 9 agents and 2 groups with weights $w_1 = w_2 = 1$.

- Group G_1 (◯): 3 agents at 0, one agent at $1/2$, one agent at 1.
- Group G_2 (□): 4 agents at 1.



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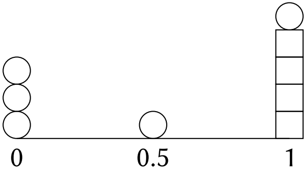

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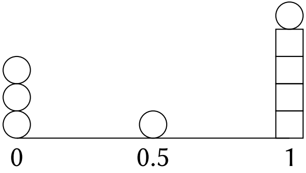

Balanced Mechanism

When $0 \leq y < 1/2$, $\max_{j \in \{1,2\}} L_j(y) = 3$, $\max_{j' \in \{1,2\}} R_{j'}(y) = 4$.
When $1/2 \leq y < 1$, $\max_{j \in \{1,2\}} L_j(y) = 4$, $\max_{j' \in \{1,2\}} R_{j'}(y) = 4$.

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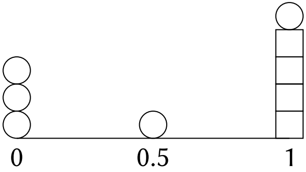

Balanced Mechanism

The facility is placed at the $1/2$, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = 2$.

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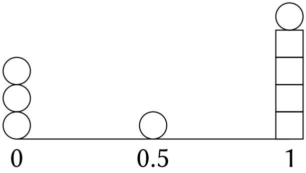

Median Mechanism

The facility is placed at the 1, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = \frac{7}{2}$.

Balanced Mechanism: Example

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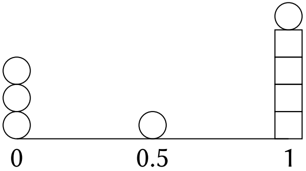
- Group G_2 (□): 4 agents at 1.

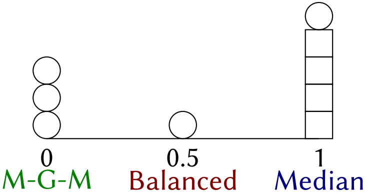


Majority Group Median Mechanism [ZLC22]

Median of Group Median Mechanism [ZLC22]

The facility is placed at the 0, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = 4$.

Balanced Mechanism: Example



- Median: facility at 1, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = 7/2$.
- Majority Group Median/Median of Group Median: facility at 0, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = 4$.
- Balanced: facility at $1/2$, $\text{mge}(\boldsymbol{\theta}, f(\boldsymbol{\theta})) = 2$ (**Optimal**).

Single-Facility: weighted Maximum Group Cost

MAJOR-PHANTOM Mechanisms[a]

[a] Phantom mechansim [Mou80]

Input: Agent profile $\boldsymbol{\theta}$, group weights $\{w_j\}_{j\in[m]}$.

1. $G_{\max}$: the largest weight group.
2. $\mathbf{x}^{G_{\max}}$: the location profile of agents in $G_{\max}$.
3. Consider $|G_{\max}| - 1$ constant values: $v_1, \ldots, v_{|G_{\max}|-1}$.
4. Place the facility at median$(\mathbf{x}^{G_{\max}}, v_1, \ldots, v_{|G_{\max}|-1})$.

Result Overview

Setting	Objectives	Mechanisms	Bounds
$k = 1$	wTGC	BALANCED	2
	wMGC	Major-Phantom	2
$k = 2$	wTGC	EndPoints	$1 + (n-2)\frac{w_{\max}}{w_{\min}}$
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$k \geq 3$	General	/	∞

Takeaways

- We study a general fairness objective, Maximum Group Effect (mge).
- We explore strategyproof deterministic mechanisms for minimizing mge and establish tight approximation bounds.
- Future directions include randomized mechanisms for mge and mechanism design in high dimensional settings.

Thank you for your attention!

Poster session: Jan 24 (Hall 4 1046)

Reference

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